1.

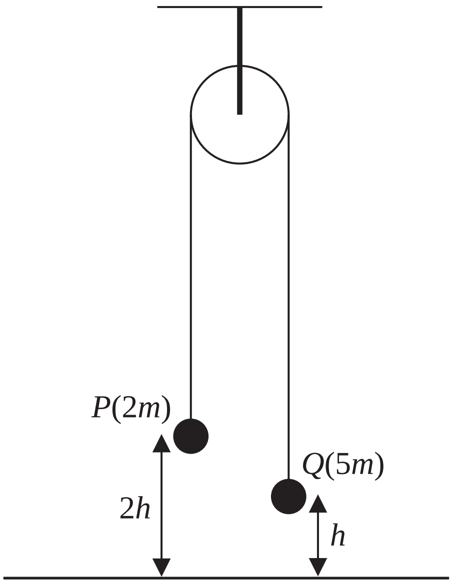


Figure 1

A ball P of mass $2m$ is attached to one end of a string.

The other end of the string is attached to a ball Q of mass $5m$.

The string passes over a fixed pulley.

The system is held at rest with the balls hanging freely and the string taut.

The hanging parts of the string are vertical with P at a height $2h$ above horizontal ground and with Q at a height h above the ground, as shown in Figure 1.

The system is released from rest.

In the subsequent motion, Q does not rebound when it hits the ground and P does not hit the pulley.

The balls are modelled as particles.

The string is modelled as being light and inextensible.

The pulley is modelled as being small and smooth.

Air resistance is modelled as being negligible.

Using this model,

(a) (i) write down an equation of motion for P,

(ii) write down an equation of motion for Q,

(4)

(b) find, in terms of h only, the height above the ground at which P first comes to instantaneous rest.

(7)

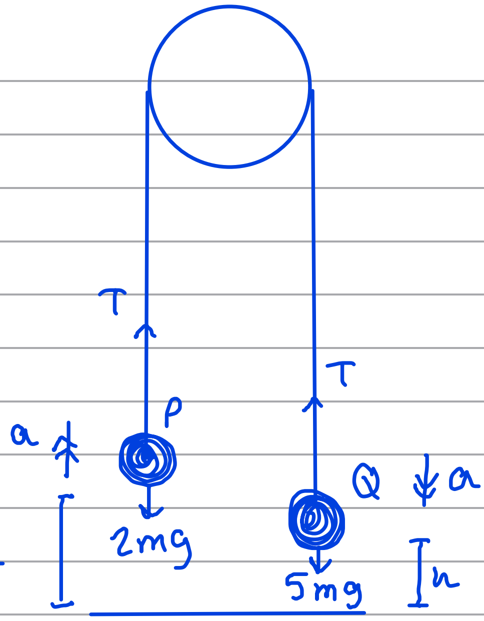
(c) State one limitation of modelling the balls as particles that could affect your answer to part (b).

(1)

In reality, the string will not be inextensible.

(d) State how this would affect the accelerations of the particles.

(1)



a) (i) Always resolve in the direction of acceleration, using $F = ma$.

For P, $R(\uparrow)$: $T - 2mg = 2ma$ ① ②

(ii)

For Q, $R(\downarrow)$: $5mg - T = 5ma$ ② ②

b) can solve ① and ② simultaneously to find acceleration

① + ②: $T - 2mg + 5mg - T = 2ma + 5ma$ ①

$3mg = 7ma$

$a = \frac{3}{7}g$ ①

When Q hits ground it has moved down h, so P has moved up h. ∴ when Q hits ground P is 3h above ground. When Q hits ground, string goes slack (no more tension) so P moves freely under gravity.

speed at which Q hits ground:

$s = h$
$u = 0$
$v = v$
$a = \frac{3g}{7}$
$t =$

"$v^2 = u^2 + 2as$"

$v^2 = 0 + 2 \times \frac{3g}{7} \times h$

$v^2 = 8.4h$ ①

$v = \sqrt{8.4h}$

distance P moves after Q hits:

$s = H$
$u = \sqrt{8.4h}$
$v = 0$
$a = -g$
$t =$

"$v^2 = u^2 + 2as$"

$0 = 8.4h - 2Hg$ ①

$H = \frac{3h}{7}$ ①

speed of P = speed of Q while string is taut.

①

total distance above ground of P $= 3h + \frac{3h}{7} = \frac{24h}{7}$ ①

c) Q will not fall exactly h metres (1)

d) the accelerations of the balls would not have equal magnitude. (1)

2.

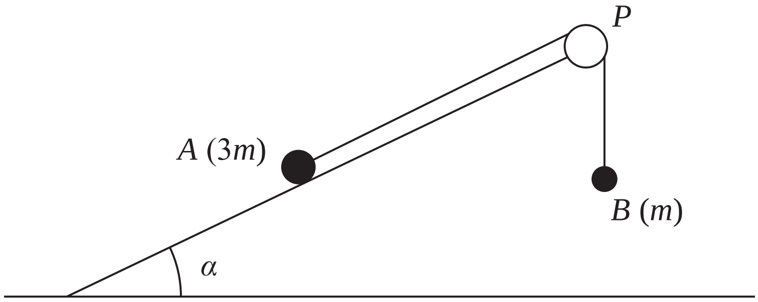


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$ $\mu = \frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

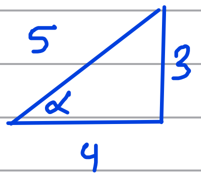
In reality, the string is not light.

(d) State how this would affect the working in part (b).

(1)

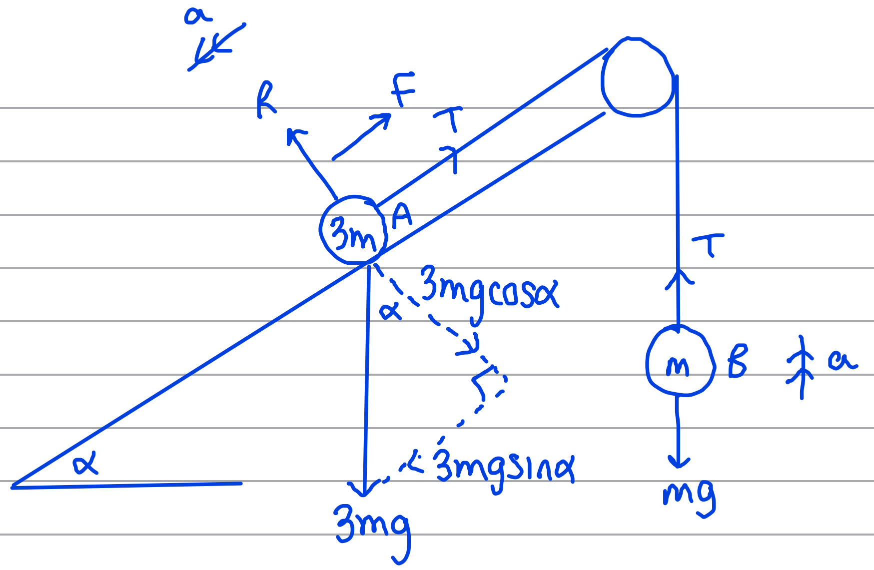
Question 2 continued

a) $\tan\alpha = 3/4$

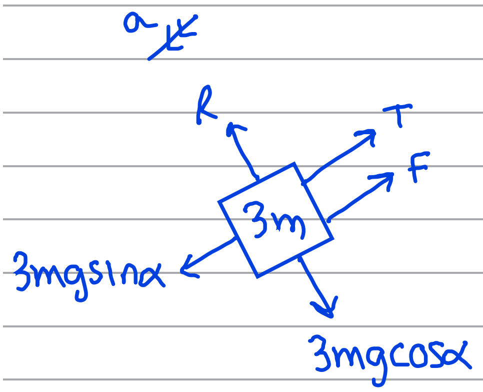


$\sin\alpha = 3/5$

$\cos\alpha = 4/5$



considering A:



1 1

$R(\swarrow): 3mg\sin\alpha - T - F = 3ma$

1

b) $R(\nwarrow)$ for A, forces balanced: $R = 3mg\cos\alpha = \frac{12mg}{5}$ 1

$F = \mu R, \ \mu = \frac{1}{6}$

$\therefore F = \frac{1}{6} \times \frac{12mg}{5} = \frac{2}{5}mg$ 1

1

$R(\uparrow)$ for B: $T - mg = ma$ 1 1

from (a), $3mg\sin\alpha - T - F = 3ma$

$$\frac{9mg}{5} - T - \frac{2mg}{5} = 3ma$$

$$\frac{7mg}{5} - T = 3ma \quad (2)$$

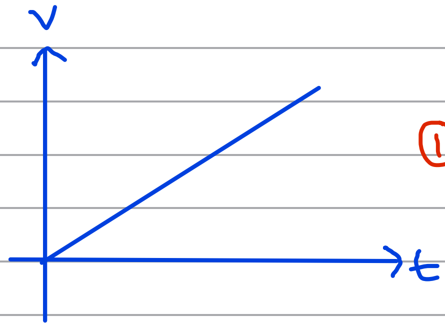
Question 2 continued

① + ②: $\frac{7mg}{5} - mg = 3ma + ma$ ①

$\frac{2}{5}mg = 4ma$

$a = \frac{1}{10}g$ as required ①

c)



①

Since $a = \frac{g}{10}$, B moves with constant acceleration, and starts from rest. ①

d) the tensions in the two equations of motion would be different ①